CONSERVATION OF ENERGY

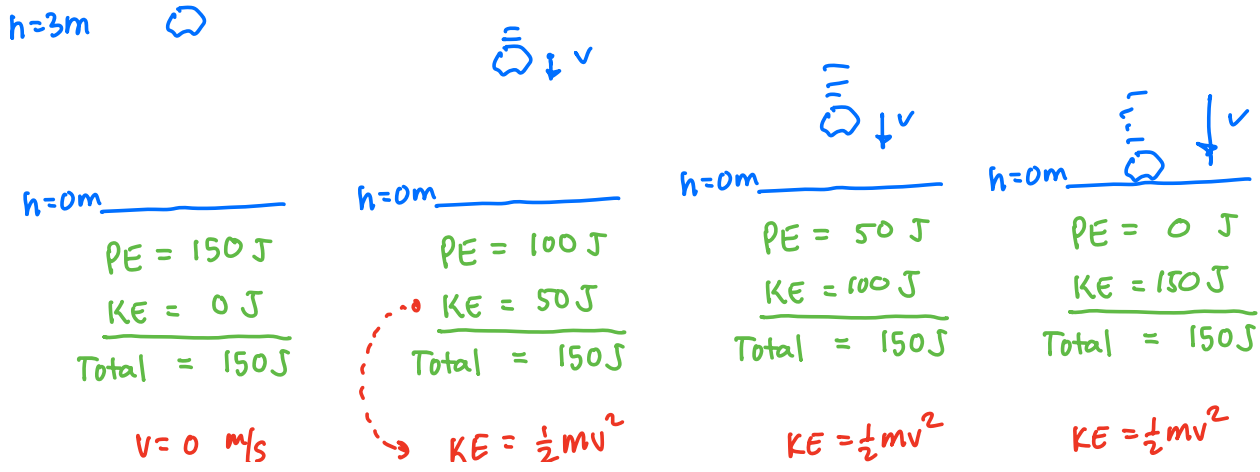
The Conservation of Energy is one of the most fundamental ideas in all of Science. Basically, it says that the total amount of energy cannot change, but the kinds of energy can change. For example, a 5 kg rock held 3 meters in the air has a potential energy of 150J.

$$PE = mgh = (5)(10)(3) = \boxed{150\,J}$$

When the rock is let go, it falls and loses potential energy because the height is getting smaller. This "lost" potential energy turns into a different kind of energy - kinetic, so that the TOTAL amount of energy doesn't change. As it loses PE it gains KE, and the sum of KE+PE always gives the same total amount. By the time the rock hits the ground, all its PE has turned into KE. [Then it crashes to a stop and the KE turns into SOUND ENERGY and THERMAL ENERGY - but we won't do math with that part.] So the 150 J of PE has turned into 150 J of KE just as it hits the ground - so the rock is moving at 7.75 m/s just as it hits the ground.

$$KE = \tfrac{1}{2}mv^2 \qquad 150 = \tfrac{1}{2}(5)v^2 \qquad v^2 = 60 \qquad \boxed{v = 7.75\ m/s}$$

The rock falling to the ground is shown below as some snapshots...



$50 = \frac{1}{2}(5)v^2$ $\quad$ $100 = \frac{1}{2}(5)v^2$ $\quad$ $150 = \frac{1}{2}(5)v^2$

$v^2 = 20$ $\quad$ $v^2 = 40$ $\quad$ $v^2 = 60$

$v = 4.47$ m/s $\quad$ $v = 6.33$ m/s $\quad$ $v = 7.75$ m/s

One way to look at the conservation of energy is to say

the total initial energy = the total final energy

same thing, written in words then symbols

$$\Sigma E_i = \Sigma E_f$$

This just means when we add up the initial KE and the initial PE we will get the same total when we add up the final KE and final PE. In symbols

$$\Sigma E_i = \Sigma E_f$$

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

i = initial
f = final

(Oftentimes, one of the velocities and one of the heights are 0, so it isn't that daunting...)

① Example:

A 4 kg ball is throw straight up with an initial speed of 15 m/s. How high does it go?

$m = 4$ kg

$v_i = 15$ m/s

$h_i = 0$ m/s

$v_f = 0$ m/s

$h_f = ?$ ← @ max height v=0!

$$\Sigma E_i = \Sigma E_f$$

$$KE_i + PE_i = KE_f + PE_f \quad *$$

$$\frac{1}{2}(4)(15)^2 + 0 = 0 + (4)(10)h_f$$

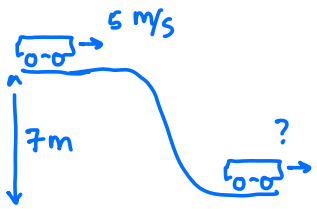
$$450 = 40h$$

$$\boxed{h = 11.25 \text{ m}}$$

* $PE_i = (4)(10)(0) = 0$

$KE_f = \frac{1}{2}(4)(0)^2 = 0$

② Example (harder)



A 300 kg coaster is moving at 5 m/s along a 7 m high platform. It then rolls down to the ground. How fast is it going at the bottom?

$m = 300$ kg
$v_i = 5$ m/s
$h_i = 7$ m

$v_f = ?$
$h_f = 0$ m

$$\Sigma E_i = \Sigma E_f$$

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}(300)(5)^2 + (300)(10)(7) = \frac{1}{2}(300)v^2 + 0$$

$$3750 + 21{,}000 = 150\,v^2$$

$$24{,}750 = 150\,v^2$$

$$v^2 = 165$$

$$\boxed{v = 12.8 \text{ m/s}}$$